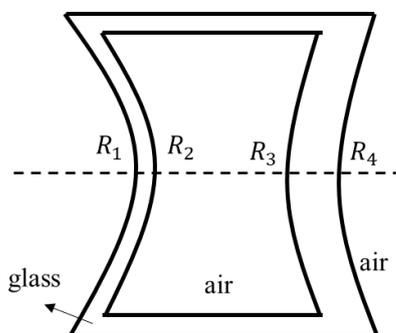


**Subject Part -1: Physics**  
**SECTION 1 (32 Marks)**

- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered).  
*Negative Marks* : -1 In all other cases.

- Q.1 Consider a nuclear process  ${}^{23}_{12}\text{Mg} \rightarrow {}^0_1\text{e} + {}^m_n\text{X}$ . Then, the values of  $m$  and  $n$  are  
 (A)  $m = 23; n = 12$  (B)  $m = 19; n = 10$  (C)  $m = 23; n = 11$  (D)  $m = 23; n = 13$
- Q.2 From the options given below, \_\_\_\_\_ is **NOT** a composite particle.  
 (A) Proton (B)  $\pi^+$  meson (C) Photon (D) Neutron
- Q.3 A barometer kept in a stationary elevator measures air pressure of 760 mm of mercury. If the elevator accelerates upwards, the barometer reading (in mm), will be  
 (A) less than 760 (B) greater than 760 (C) equal to 760 (D) zero
- Q.4 A lens system is formed from glass in the shape as shown in the figure. The refractive index of the glass is  $n$ , and the radii of curvature of the four curved surfaces as indicated are  $R_1 = R_2 = R$  and  $R_3 = R_4 = 2R$ .



The effective focal length of the lens system is

- (A)  $\frac{R}{(n-1)}$  (B)  $\frac{2R}{(n-1)}$  (C) zero (D) infinite

- Q.5 If  $v_{rms}$  denotes the root mean square speed of atoms of an ideal monoatomic gas and  $T$  denotes the absolute temperature, then according to the kinetic theory of an ideal gas
- (A)  $v_{rms} \propto T^{1/2}$       (B)  $v_{rms} \propto T$       (C)  $v_{rms} \propto T^{3/2}$       (D)  $v_{rms} \propto T^2$
- Q.6 A constant electrical power is being transmitted at a particular voltage ( $V_0$ ) through cables of constant resistance. If  $V_0$  is doubled, the correct option for power loss ( $P_L$ ) in transmission is
- (A)  $P_L$  decreases by a factor of 4.      (B)  $P_L$  decreases by a factor of 2.  
(C)  $P_L$  increases by a factor of 4.      (D)  $P_L$  increases by a factor of 2.
- Q.7 A particle of mass 1 kg moves in the  $xy$  plane under the action of a force  $\vec{F}$  that depends on the coordinates of the point as  $\vec{F} = (0.12y\hat{x} + 2\hat{y})$  N, where  $\hat{x}$  and  $\hat{y}$  represent unit vectors along the  $+x$  and  $+y$  directions, respectively. The particle is at rest at location (0, 0) m at time  $t = 0$  s. The component of the velocity of the particle (in  $\text{m s}^{-1}$ ) in  $x$ -direction at time  $t = 10$  s is
- (A) 4      (B) 40      (C) 6      (D) 60
- Q.8 If the electric field is given by  $\vec{E} = \frac{K}{r} \hat{r}$ , where  $r$  is the radial distance and  $\hat{r}$  represents the unit vector along  $r$ , then the unit of  $K$  is
- (A)  $\frac{\text{Nm}}{\text{C}^2}$       (B)  $\frac{\text{N}}{\text{C}}$       (C)  $\frac{\text{Nm}^2}{\text{C}}$       (D)  $\frac{\text{Nm}}{\text{C}}$

**Subject Part -1: Physics**  
**SECTION 2 (24 Marks)**

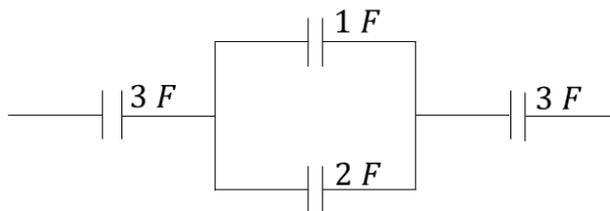
- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

Q.9 The binding energy and mass of  ${}^p_qX$  are  $54 \times 10^{-13}$  J and  $1.33 \times 10^{-26}$  kg, respectively. If  $p = 2q$ , the value of  $q$  is \_\_\_\_\_.

[Take mass of a proton as well as the mass of a neutron as  $1.67 \times 10^{-27}$  kg, and speed of light in vacuum as  $3 \times 10^8$  m s<sup>-1</sup>]

Q.10 An aluminum ball weighing 10 g is heated to 225 °C and then dropped into 100 g of a liquid, initially at 20 °C. The ball and the liquid reach a new equilibrium temperature of 25 °C. The specific heat capacity of the liquid in this temperature range is  $4 \text{ kJ kg}^{-1} \text{ K}^{-1}$ . If no heat is lost to the surroundings during this process, the specific heat capacity (in  $\text{kJ kg}^{-1} \text{ K}^{-1}$ ) of aluminum ball is \_\_\_\_\_.

Q.11 Consider the combination of capacitors as shown in the figure below:



The equivalent capacitance (in F) of the combination is \_\_\_\_\_.

Q.12 A particle is moving in the  $xy$  plane. When it passes through the point (3, 1) m, its angular momenta about the origin and the point (1, -1) m are  $10 \text{ kg m}^2 \text{ s}^{-1}$  and  $12 \text{ kg m}^2 \text{ s}^{-1}$ , respectively. The magnitude of the angular momentum (in  $\text{kg m}^2 \text{ s}^{-1}$ ) of the particle about the point (2, 1) m is \_\_\_\_\_.

- Q.13 A square conducting loop of side 1 m and resistance  $1 \Omega$ , is moving with velocity  $1 \hat{x} \text{ m s}^{-1}$  through a uniform magnetic field  $\vec{B} = 1 \hat{z}$  Tesla, where  $\hat{x}$  and  $\hat{z}$  represent unit vectors along the  $+x$  and  $+z$  directions, respectively. If  $\frac{\pi}{3}$  is the angle between the normal to the plane of the loop and the magnetic field, then the current (in Ampere) in the loop at  $t = 1 \text{ s}$  is \_\_\_\_\_.
- Q.14 A uniform string of length 1 m is fixed at its two ends. The mass of the string is 1 g and tension in it is 10 N. If the string is made to vibrate with frequency 500 Hz, then the number of nodes generated between the two fixed ends of the string is \_\_\_\_\_.

**Subject Part -1: Physics**  
**SECTION 3 (24 Marks)**

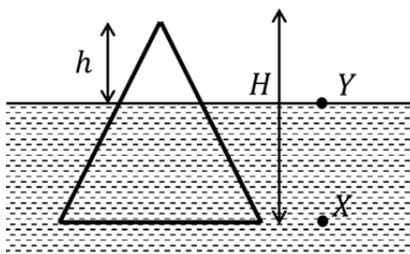
- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
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<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct option is chosen;
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- The answer to the **SECOND** question is a NUMERICAL VALUE.  
If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.
  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme

<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct numerical value is entered;
<i>Zero Marks</i>	:	0	In all other cases.

**PARAGRAPH I**

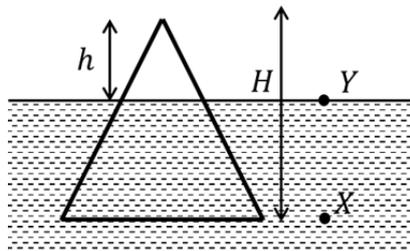
A conical iceberg of total height  $H$  is floating in still sea water, as shown in the figure below. The height of the iceberg above the water surface is  $h$ . The density of ice is  $\rho_{ice} = 924 \text{ kg m}^{-3}$  and that of sea water is  $\rho_w = 1056 \text{ kg m}^{-3}$ .



- Q.15 The pressure difference between points  $X$  (at the bottom of the iceberg) and  $Y$  (at the water surface) is
- (A)  $\rho_w g H$                       (B)  $\rho_w g (H - h)$                       (C)  $\rho_{ice} g (H - h)$                       (D)  $\rho_w g h$

**PARAGRAPH I**

A conical iceberg of total height  $H$  is floating in still sea water, as shown in the figure below. The height of the iceberg above the water surface is  $h$ . The density of ice is  $\rho_{ice} = 924 \text{ kg m}^{-3}$  and that of sea water is  $\rho_w = 1056 \text{ kg m}^{-3}$ .



- Q.16 Neglecting the density of air, the ratio of the total height of the iceberg to that of its height above the water surface,  $\frac{H}{h}$ , is \_\_\_\_\_.

**PARAGRAPH II**

The electromagnetic waves propagating in free space carry energy density given by  $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$ , where  $E$  and  $B$  are magnitudes of the electric and magnetic field components and  $\epsilon_0$  and  $\mu_0$  are the free space permittivity and permeability, respectively. The speed of light in free space is  $c = 3 \times 10^8 \text{ m s}^{-1}$ . The vectors corresponding to the electric and magnetic fields, and the wave propagation are mutually orthogonal.

- Q.17 Regarding the properties of the electromagnetic waves, the correct statement from the following is
- (A) Electric and magnetic energy densities are equal.
  - (B) Electric energy density is larger than the magnetic energy density by a factor of  $9 \times 10^{16}$ .
  - (C) Electric energy density is smaller than the magnetic energy density by a factor of  $9 \times 10^{16}$ .
  - (D) Electric energy density is double the magnetic energy density.

**PARAGRAPH II**

The electromagnetic waves propagating in free space carry energy density given by  $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$ , where  $E$  and  $B$  are magnitudes of the electric and magnetic field components and  $\epsilon_0$  and  $\mu_0$  are the free space permittivity and permeability, respectively. The speed of light in free space is  $c = 3 \times 10^8 \text{ m s}^{-1}$ . The vectors corresponding to the electric and magnetic fields, and the wave propagation are mutually orthogonal.

- Q.18 The electric field vector of an electromagnetic wave is given by  $\vec{E} = E_0 \sin(10^7 x + 3 \times 10^{15} t)(6\hat{y} + 2\hat{z})$ . If the magnetic field component is expressed as  $\vec{B} = \frac{2E_0}{c} \sin(10^7 x + 3 \times 10^{15} t)(\hat{y} - p\hat{z})$ , then the value of  $p$  is \_\_\_\_\_.
- [ $\hat{y}$ ,  $\hat{z}$  are unit vectors along  $+y$ ,  $+z$  directions, respectively]

**PARAGRAPH III**

A resistance temperature detector (RTD) measures the temperature based on the linear dependence of resistance of a platinum wire on temperature. The RTD is calibrated by measuring resistance at 0 °C and 100 °C. The measured resistance at 0 °C is 100  $\Omega$  and that at 100 °C is 140  $\Omega$ .

- Q.19 The physical law that allows the use of resistance of RTD as a measure of temperature is
- (A) Zeroth law of thermodynamics                      (B) First law of thermodynamics  
(C) Second law of thermodynamics                      (D) Third law of thermodynamics

**PARAGRAPH III**

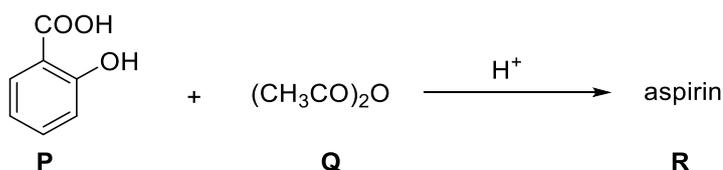
A resistance temperature detector (RTD) measures the temperature based on the linear dependence of resistance of a platinum wire on temperature. The RTD is calibrated by measuring resistance at 0 °C and 100 °C. The measured resistance at 0 °C is 100  $\Omega$  and that at 100 °C is 140  $\Omega$ .

- Q.20 If RTD's resistance is 102.8  $\Omega$ , the temperature (in °C) is \_\_\_\_\_.

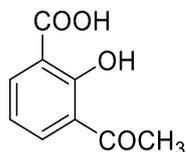
**Subject Part -2: Chemistry**  
**SECTION 1 (32 Marks)**

- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
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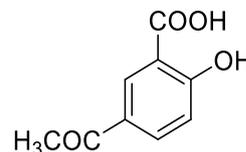
Q.21 Salicylic acid **P** reacts with acetic anhydride **Q** to give aspirin **R** as the product. The correct structure of aspirin is



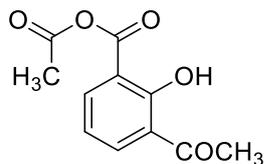
(A)



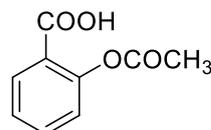
(B)



(C)



(D)



Q.22 The one that has the lowest second ionization enthalpy among the following is

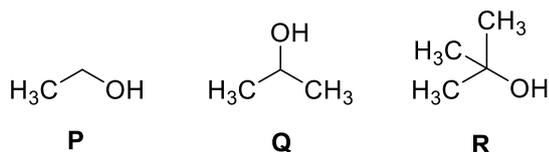
(A) Ca

(B) Be

(C) Ba

(D) Sr

Q.23 The acid strength of the alcohols given below follows the order



(A)  $\text{R} > \text{Q} > \text{P}$

(B)  $\text{P} > \text{Q} > \text{R}$

(C)  $\text{Q} > \text{R} > \text{P}$

(D)  $\text{R} > \text{P} > \text{Q}$

Q.24 The compound that has an unpaired electron is

(A)  $\text{D}_2\text{O}$

(B)  $\text{KO}_2$

(C)  $\text{Na}_2\text{O}$

(D)  $\text{Na}_2\text{O}_2$

Q.25 The correct statement among the following is

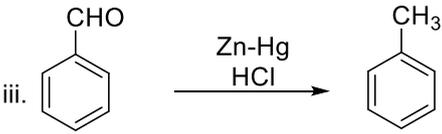
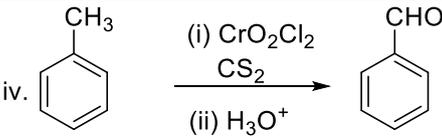
(A) HF is a stronger acid than HCl

(B)  $\text{NaNH}_2$  is a stronger base than NaOH

(C)  $\text{HNO}_2$  is a stronger acid than  $\text{HNO}_3$

(D)  $\text{Na}_2\text{CO}_3$  is a stronger base than NaOH

Q.26 Match the reactions given in **Column-I** with the corresponding type/name of the reaction given in **Column-II** and choose the correct option.

Column-I		Column-II
Reaction		Type/name
i.	$\text{CH}_3\text{CHO} \xrightarrow[\Delta]{\text{dil. NaOH}} \text{CH}_3-\text{CH}=\text{CH}-\text{CHO}$	p. Cannizzaro reaction
ii.	$\text{HCHO} \xrightarrow[\Delta]{\text{conc. NaOH}} \text{CH}_3\text{OH} + \text{HCOONa}$	q. Clemmensen reduction
iii.		r. Etard reaction
iv.		s. Aldol condensation

(A)  $i \rightarrow s, ii \rightarrow p, iii \rightarrow r, iv \rightarrow q$

(B)  $i \rightarrow s, ii \rightarrow p, iii \rightarrow q, iv \rightarrow r$

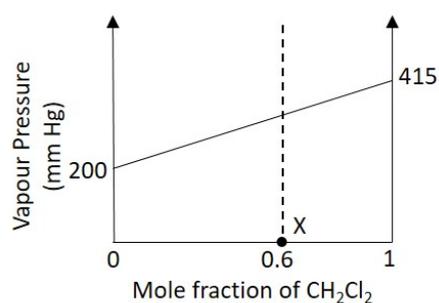
(C)  $i \rightarrow p, ii \rightarrow s, iii \rightarrow q, iv \rightarrow r$

(D)  $i \rightarrow p, ii \rightarrow s, iii \rightarrow r, iv \rightarrow q$

Q.27 The correct statement among the following is

- (A) An orbital of an atom can accommodate up to four electrons.
- (B) Electrons present in an orbital of an atom must have opposite spins.
- (C) Two electrons in an atom can have the same set of four quantum numbers.
- (D) Energy of  $3p$  orbitals of an atom is higher than that of  $3d$  orbitals.

Q.28 Chloroform ( $\text{CHCl}_3$ ) and dichloromethane ( $\text{CH}_2\text{Cl}_2$ ) at 298 K are mixed in a vessel. They form an ideal solution. The plot of vapour pressure versus mole fraction of  $\text{CH}_2\text{Cl}_2$  is given below. The solution with composition corresponding to point X in the figure has the vapour pressure (in mm Hg) of



- (A) 308                      (B) 615                      (C) 186                      (D) 329

**Subject Part -2: Chemistry**  
**SECTION 2 (24 Marks)**

- This section contains **Six (6)** questions.
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- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
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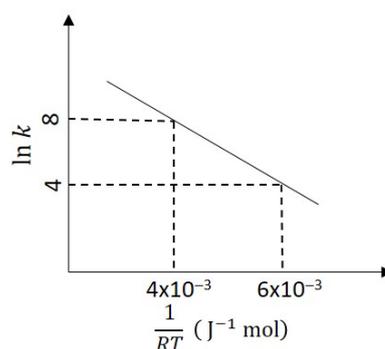
Q.29 Vinyl chloride is polymerized to give 312.5 g of polyvinylchloride. If the reaction takes place with 100% conversion, the number of moles of vinyl chloride used is \_\_\_\_\_.

[Given: Molar mass of H, C and Cl are 1, 12, and 35.5 g mol<sup>-1</sup>, respectively]

Q.30 The number of unpaired electrons present in the ground state electronic configuration of Fe is \_\_\_\_\_.

[Given: Atomic number of Fe = 26]

Q.31 The plot of  $\ln k$  versus  $\frac{1}{RT}$  for a reaction is shown in the following figure. The activation energy (in kJ mol<sup>-1</sup>) of the reaction is \_\_\_\_\_.

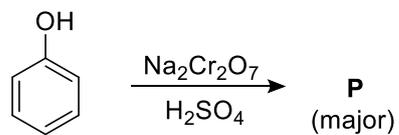


Q.32 The wavelength of light absorbed by a solution is 600 nm. The energy of the photon absorbed (in J) is  $x \times 10^{14} \times h$ , where  $x$  is \_\_\_\_\_.

[  $h$  is Planck's constant; speed of light in vacuum =  $3 \times 10^8$  m s<sup>-1</sup> ]

Q.33 In an isothermal reversible process at 50 K, a system absorbs heat of  $250 \text{ J mol}^{-1}$ . The change in entropy of the system (in  $\text{J mol}^{-1} \text{ K}^{-1}$ ) is \_\_\_\_\_.

Q.34 The total number of oxygen atoms present in the major product **P** of the reaction given below is \_\_\_\_\_.



**Subject Part -2: Chemistry**  
**SECTION 3 (24 Marks)**

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
  - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
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<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct option is chosen;
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- The answer to the **SECOND** question is a NUMERICAL VALUE.
 

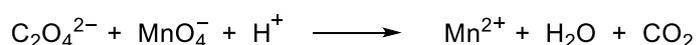
If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.

  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme

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**PARAGRAPH IV**

In an aqueous acidic medium, oxalate ion ( $\text{C}_2\text{O}_4^{2-}$ ) is oxidized by permanganate ion ( $\text{MnO}_4^-$ ) to produce  $\text{CO}_2$  and  $\text{Mn}^{2+}$ . The equation for this reaction (not balanced) is given below.

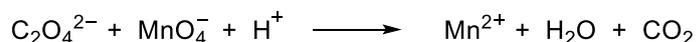


Q.35 The number of *d*-electron(s) present in the central metal of  $\text{MnO}_4^-$  is

- (A) 0                      (B) 2                      (C) 5                      (D) 7

**PARAGRAPH IV**

In an aqueous acidic medium, oxalate ion ( $\text{C}_2\text{O}_4^{2-}$ ) is oxidized by permanganate ion ( $\text{MnO}_4^-$ ) to produce  $\text{CO}_2$  and  $\text{Mn}^{2+}$ . The equation for this reaction (not balanced) is given below



Q.36 The number of moles of  $\text{CO}_2$  produced using one mole of  $\text{MnO}_4^-$  is \_\_\_\_\_.

**PARAGRAPH V**

Standard electrode potential is used to decide the reducing and oxidizing power of metals.

[Given:  $E^\ominus(\text{H}^+/\text{H}_2) = 0 \text{ V}$ ,  $E^\ominus(\text{Pb}^{2+}/\text{Pb}) = -0.13 \text{ V}$ ,  $E^\ominus(\text{Ni}^{2+}/\text{Ni}) = -0.25 \text{ V}$ ,  $E^\ominus(\text{Zn}^{2+}/\text{Zn}) = -0.76 \text{ V}$ ,  
 $E^\ominus(\text{Cu}^{2+}/\text{Cu}) = 0.34 \text{ V}$ ]

Q.37 Which of the following metals **DOES NOT** dissolve in HCl?

- (A) Ni                      (B) Pb                      (C) Cu                      (D) Zn

**PARAGRAPH V**

Standard electrode potential is used to decide the reducing and oxidizing power of metals.

[Given:  $E^\ominus(\text{H}^+/\text{H}_2) = 0 \text{ V}$ ,  $E^\ominus(\text{Pb}^{2+}/\text{Pb}) = -0.13 \text{ V}$ ,  $E^\ominus(\text{Ni}^{2+}/\text{Ni}) = -0.25 \text{ V}$ ,  $E^\ominus(\text{Zn}^{2+}/\text{Zn}) = -0.76 \text{ V}$ ,  
 $E^\ominus(\text{Cu}^{2+}/\text{Cu}) = 0.34 \text{ V}$ ]

Q.38 For a cell reaction,  $\text{Zn(s)} + \text{Cu}^{2+} (0.01 \text{ M}) \rightarrow \text{Zn}^{2+} (0.0001 \text{ M}) + \text{Cu(s)}$ ,  $E_{\text{cell}}$  (in V) at 298 K is \_\_\_\_\_.

[ Use  $(2.303 \text{ RT/F}) = 0.059 \text{ V}$  at 298 K]

**PARAGRAPH VI**

Aniline reacts with bromine water at room temperature to give 2,4,6-tribromoaniline.

Q.39 Choose the correct option with regard to the above reaction.

- (A)  $\text{FeBr}_3$  is required for this reaction.  
(B) Bromine radical is formed as an intermediate.  
(C) Addition of HBr increases the rate of this reaction.  
(D) This is an example of electrophilic aromatic substitution reaction.

**PARAGRAPH VI**

Aniline reacts with bromine water at room temperature to give 2,4,6-tribromoaniline.

Q.40 The minimum number of moles of  $\text{Br}_2$  required to react completely with 2 moles of aniline in the above reaction is \_\_\_\_\_.

**Subject Part -3: Mathematics**  
**SECTION 1 (32 Marks)**

- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:  
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*Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered).  
*Negative Marks* : -1 In all other cases.

Q.41 Consider an equilateral triangle inscribed in the unit circle centered at  $(0, 0)$ . If one of the vertices of the triangle is  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ , then which one of the following is also a vertex of this triangle?

- (A)  $\left(-\frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$  (B)  $\left(-\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}}\right)$  (C)  $\left(-\frac{\sqrt{3}+1}{\sqrt{2}}, \frac{\sqrt{3}-1}{\sqrt{2}}\right)$  (D)  $\left(-\frac{\sqrt{3}+1}{2\sqrt{2}}, -\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

Q.42 Let  $X$  and  $Y$  be two events in a sample space. Let  $X^c$  and  $Y^c$  denote the complement events of  $X$  and  $Y$ , respectively. If  $X$  and  $Y$  are independent, then which one of the following options is true ?

- (A)  $X$  and  $Y$  are mutually exclusive (B)  $P(X) = P(Y)$   
 (C)  $P(X) + P(Y) = 1$  (D)  $P(X^c \cap Y^c) = (1 - P(X))(1 - P(Y))$

Q.43 Let  $X$  and  $Y$  be two events such that  $P(X|Y) = \frac{1}{3}$ ,  $P(Y|X) = \frac{1}{4}$  and  $P(X) = \frac{1}{3}$ . Let  $X^c, Y^c$  denote the complement events of  $X, Y$ , respectively.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
i. $P(Y)$	p. $\frac{3}{4}$
ii. $P(X^c Y)$	q. $\frac{1}{4}$
iii. $P(Y^c X^c)$	r. $\frac{2}{3}$
iv. $P(X \cap Y)$	s. $\frac{1}{12}$

- (A) i  $\rightarrow$  q, ii  $\rightarrow$  r, iii  $\rightarrow$  p, iv  $\rightarrow$  s  
 (B) i  $\rightarrow$  p, ii  $\rightarrow$  r, iii  $\rightarrow$  q, iv  $\rightarrow$  s  
 (C) i  $\rightarrow$  r, ii  $\rightarrow$  s, iii  $\rightarrow$  p, iv  $\rightarrow$  q  
 (D) i  $\rightarrow$  q, ii  $\rightarrow$  r, iii  $\rightarrow$  s, iv  $\rightarrow$  p

Q.44 For a real number  $a$ , consider the following system of linear equations in variables  $x, y, z$ :

$$4ax + y + az = 0$$

$$x + a^2y = 0$$

$$ax + z = 0$$

Then the number of values of  $a$ , for which the system has infinitely many solutions, is \_\_\_\_\_.

- (A) 2                      (B) 0                      (C) 4                      (D) infinitely many

Q.45 The area bounded by the curves  $y = e^{-|x|}$  and  $y = e^{|x|-2}$  is \_\_\_\_\_.

- (A)  $2e^2(e^2 - 2e + 1)$                       (B)  $\frac{2(e^2 - 2e + 1)}{e^2}$   
 (C)  $2e(e^2 - 2e + 1)$                       (D)  $\frac{2(e^2 - 2e + 1)}{e}$

Q.46 The value of  $p$ , for which

$$\int_1^4 \frac{p}{x(16 - x^{\frac{3}{2}})} dx = \log_e 15,$$

is \_\_\_\_\_.

- (A) 24                      (B) 23                      (C) 22                      (D) 25

Q.47 Consider the function  $f$  defined on the real line given by

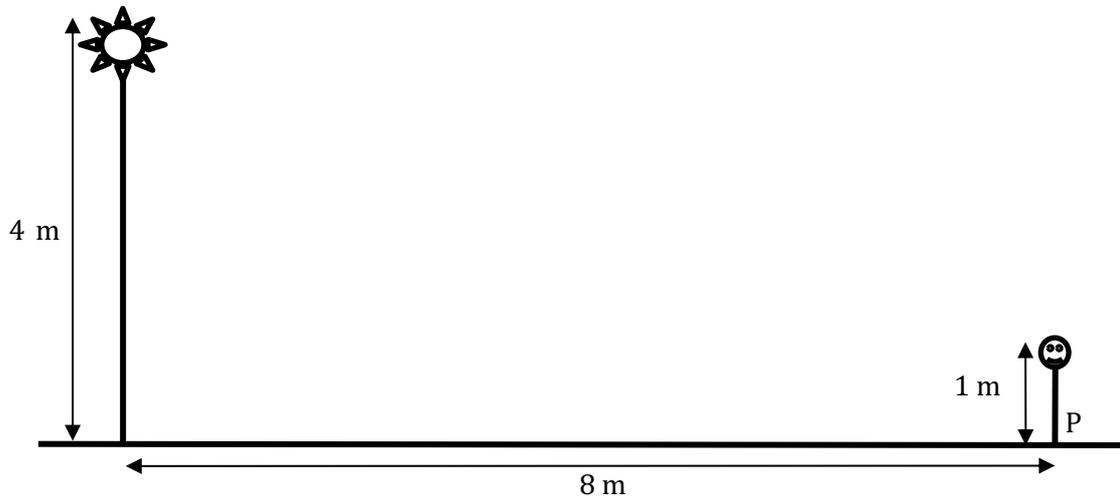
$$f(x) = \tan^{-1}(e^x - x^2),$$

where  $\tan^{-1} \theta$  is the principal value branch of the inverse function of  $\tan \theta$ .

Then the range of  $f$  is \_\_\_\_\_.

- (A)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$                       (B)  $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$                       (C)  $\left[0, \frac{\pi}{4}\right]$                       (D)  $(-\infty, \infty)$

- Q.48 A child of height 1 meter starts running from the point P towards a glowing lamp post which is of height 4 meter and is at 8 meter distance from the point P. If the child is running with a constant speed of 1 meter per second, then after how many seconds will the shadow of the child's head reach the point P ?



- (A) 1                      (B) 2                      (C) 3                      (D) 4

**Subject Part -3: Mathematics**  
**SECTION 2 (24 Marks)**

- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

Q.49 For a real number  $\alpha$ , if the limit

$$\lim_{x \rightarrow 0} \frac{\cos \alpha x - e^{-2\alpha x^2}}{x^2} = 2,$$

then the value of  $\alpha$  is \_\_\_\_\_.

Q.50 For real numbers  $\alpha, \beta$ , consider the function

$$f(x) = \alpha + e^{-x} + \beta x^2 \log_e x, \quad x > 0$$

If  $f(1) = 1$  and  $f'(1) = 2$ , then the value of  $\alpha + \beta$  is \_\_\_\_\_.

Q.51 Let  $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $b > a > 0$ ) be an ellipse such that the point  $(\frac{4}{5}, \sqrt{21})$  lies on  $E$ . If the length of a latus rectum is  $\frac{8}{5}$ , then the value of  $b - a^2$  is \_\_\_\_\_.

Q.52 Let  $\alpha = 4 + 2\sqrt{3}$  and  $\beta = \alpha + \frac{1}{\alpha}$ . Suppose  $\beta$  is a root of the equation  $4x^2 + px + q = 0$ , where  $p, q$  are rational numbers. Then the value of  $\frac{p+q}{11}$  is \_\_\_\_\_.

Q.53 Let  $n!$  denote the factorial of the natural number  $n$ . Let  $D$  be the determinant of the matrix

$$\begin{bmatrix} 3! & 4! & 5! \\ 4! & 5! & 6! \\ 5! & 6! & 7! \end{bmatrix}$$

Then the sum of the digits of the number  $\frac{D}{216}$  is \_\_\_\_\_.

Q.54 Let  $X$  and  $Y$  be two independent events such that  $P(X) = \frac{1}{3}$  and  $P(Y) = \frac{1}{4}$ . Then the value of  $3P(X|(X \cup Y))$  is \_\_\_\_\_.

**Subject Part -3: Mathematics**  
**SECTION 3 (24 Marks)**

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
  - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
  - Answer to each question will be evaluated according to the following marking scheme:
 

<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct option is chosen;
<i>Zero Marks</i>	:	0	In all other cases.
- The answer to the **SECOND** question is a NUMERICAL VALUE.
- If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.
  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme

<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct numerical value is entered;
<i>Zero Marks</i>	:	0	In all other cases.

**PARAGRAPH VII**

For given real numbers  $a$  and  $b$ , consider the system of equations in  $x$  and  $y$

$$\begin{aligned} ax + by &= 1 \\ x + 3y &= a \end{aligned}$$

Q.55 For which one of the following pairs  $(a, b)$ , the system has a unique solution?

(A) (1, 3)

(B) (2, 6)

(C)  $(-1, -3)$

(D) (6, 2)

**PARAGRAPH VII**

For given real numbers  $a$  and  $b$ , consider the system of equations in  $x$  and  $y$

$$\begin{aligned} ax + by &= 1 \\ x + 3y &= a \end{aligned}$$

Q.56 If the system has infinitely many solutions, then the value of  $ab$  is \_\_\_\_\_.

**PARAGRAPH VIII**

Consider the function  $f$  defined on the real line by

$$f(x) = 2x e^{-(x^2 - \frac{1}{2})}$$

Q.57 The value of

$$\int_0^{1/\sqrt{2}} f(x) dx$$

is \_\_\_\_\_.

(A)  $\sqrt{e} - 1$

(B)  $\sqrt{e} + 1$

(C)  $\sqrt{e} - 2$

(D)  $\sqrt{e} + 2$

**PARAGRAPH VIII**

Consider the function  $f$  defined on the real line by

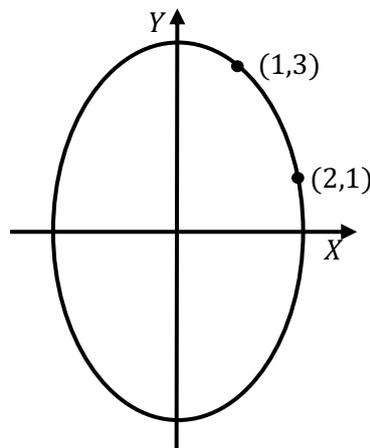
$$f(x) = 2x e^{-(x^2 - \frac{1}{2})}$$

Q.58 The maximum value of  $f$  is \_\_\_\_\_.

**PARAGRAPH IX**

Let  $a, b$  be real numbers such that the points  $(2,1)$  and  $(1,3)$  lie on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Q.59 The eccentricity of the ellipse is \_\_\_\_\_.

(A)  $\sqrt{\frac{5}{8}}$

(B)  $\sqrt{\frac{3}{8}}$

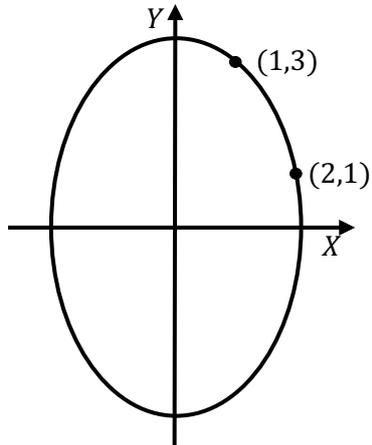
(C)  $\sqrt{\frac{3}{5}}$

(D)  $\sqrt{\frac{5}{3}}$

**PARAGRAPH IX**

Let  $a, b$  be real numbers such that the points  $(2,1)$  and  $(1,3)$  lie on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Q.60 If the distance of either of the foci from the origin is  $c$  unit, then  $\frac{12}{25}c^2$  is \_\_\_\_\_.

**END OF THE QUESTION PAPER**

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Space for rough work

Space for rough work